All the abstracts in alphabetical order

(v. 16.06.2022)

Location:	UAM	UAM	UAM	UAM	ICMAT
Time/Day	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
10:00–10:50	*Opening* (10:40–10:50)	Glaudo	Fernandez-Real	Nowak	Franceschini
10:50–11:20	Vázquez (10:50–11:40)	*Break*	*Break*	*Break*	*Break*
11:20–12:10	*Break* (11:40–12:10)	Audrito	Meglioli	de Filippis	Ibarrondo
12:10–13:00	Stan	Andreianov	Jakobsen	Volzone	del Mar
13:00–15:00	*Lunch*	*Lunch*	*Lunch*	*Lunch*	Closing (13:00–13:10)
15:00–15:50	Medina	Cinti	Grillo	Weidner	*Lunch* (13:10–15:10)
15:50–16:40	Chan	Muratori	Punzo	Chlebicka	
16:40–17:10	*Break*	*Break*	*Break*	*Break*	
17:10–18:00	Restrepo	Ispizua	Ghosh	Kim	
20:30–23:59			*Social dinner*		

Speaker: Boris Andreianov

Title: *Obtaining and using Kato inequalities for (convection-)diffusion problems*

Time slot: *Tuesday, 12:10–13:00*

Abstract:

Kato inequalities (the original version of Kato'72 being $\mathrm{sign}(W)\,\Delta W \leq \Delta|W|$ in $\mathcal{D}'$) in the context of local or non-local convection-diffusion problems

$$\partial_t u + \mathrm{div} f(u) + (-\Delta)^s \varphi(u) = 0$$

express the localized L^1 contraction principle. They appear as a key argument within uniqueness proofs. As a key example, for $\varphi \equiv 0$ the Kato inequalities are encrypted in the Kruzhkov'70 entropy inequalities, their deduction requires the celebrated doubling of variable technique. In the case of Lipschitz nonlinearities, the appropriate choice of test functions implies uniqueness of entropy solutions; but uniqueness may fail in the non-Lipschitz case as demonstrated by Panov and Kruzhkov'91. In the degenerate parabolic case, Carrillo'99 pointed out the essential role of quantifying the parabolic dissipation on the way from the entropy formulation to the Kato inequality; it can be connected to the sharp description of the dissipation measure within the kinetic formulation introduced by Chen and Perthame'2003.

Inspired by these ideas and the difficulties specific to convection-dominated conservation laws, we will discuss the obtention and the exploitation of Kato inequalities, in the context of the above non-local evolution equation [1, 2] and in the context of L^∞ solutions to the elliptic problem

$$u - \Delta\varphi(x, u) = g$$

([3]) for low-regularity nonlinearities f, φ.

Exploring the path from the Kato inequalities towards uniqueness, we highlight specific choices of test functions (see [4] for the introduction of a systematic approach of the Holmgren kind) and a set of techniques exploiting in various ways the moduli of continuity of the nonlinearities ([2, 3]).

References

[1] N. Alibaud, B. Andreianov, A. Ouédraogo. Nonlocal dissipation measure and L^1 kinetic theory for fractional conservation laws. *Comm. PDEs* 45 (2020), no. 9, 1213–1251.

[2] B. Andreianov, M. Brassart. Uniqueness of entropy solutions to fractional conservation laws with "fully infinite" speed of propagation. *J. Differ. Equ.* 268 (2020), no. 7, 3903–3935.

[3] B. Andreianov, M. Maliki. On classes of well-posedness for quasilinear diffusion equations in the whole space. *DCDS Ser. S* 14 (2021), no. 2, 505–531.

[4] J. Endal, E.R. Jakobsen. L^1 contraction for bounded (nonintegrable) solutions of degenerate parabolic equations. *SIAM J. Math. Anal.* 46 (2014), no. 6, 3957–3982.

Speaker: Alessandro Audrito

Title: *The parabolic obstacle problem with fully nonlinear diffusion*

Time slot: *Tuesday, 11:20–12:10*

Abstract:

In this talk I will present some recent results concerning the regularity and the structure of the free boundary for the parabolic obstacle problem with fully nonlinear diffusion. Our main result states the free boundary can be written as the disjoint union of a regular part, which is locally the graph of a smooth function, and a singular part, which is contained in a Lipschitz manifold. This is a joint work with X. Ros-Oton (UB) and T. Kukuljan (UB).

Speaker: Hardy Chan

Title: *Singular solutions for fractional parabolic boundary value problems*

Time slot: *Monday, 15:50–16:40*

Abstract:

The standard problem for the classical heat equation posed in a bounded domain Ω of $\mathbb{R}^n$ is the initial and boundary value problem. If the Laplace operator is replaced by a version of the fractional Laplacian, the initial and boundary value problem can still be solved on the condition that the non-zero boundary data must be singular, i.e., the solution $u(t,x)$ blows up as x approaches $\partial\Omega$ in a definite way. In this paper we construct a theory of existence and uniqueness of solutions of the parabolic problem with singular data taken in a very precise sense, and also admitting initial data and a forcing term. When the boundary data are zero we recover the standard fractional heat semigroup. A general class of integro-differential operators may replace the classical fractional Laplacian operators, thus enlarging the scope of the work. As further results on the spectral theory of the fractional heat semigroup, we show that a one- sided Weyl-type law holds in the general class, which was previously known for the restricted and spectral fractional Laplacians, but is new for the censored (or regional) fractional Laplacian. This yields bounds on the fractional heat kernel. We also mention some ongoing work for the corresponding time-fractional equation. This is a joint work with David Gomez-Castro and Juan Luis Vazquez.

Speaker: Iwona Chlebicka

Title: *Potential estimates for solutions to quasilinear elliptic problems with general growth and regularity consequences*

Time slot: *Thursday, 15:50–16:40*

Abstract:

We consider measure data elliptic problems involving a second order operator exhibiting Orlicz growth and having measurable coefficients. As known in the p-Laplace case, pointwise estimates for solutions expressed with the use of nonlinear potential are powerful tools in the study of the local behaviour of the solutions. Not only we provide such estimates expressed in terms of a potential of generalized Wolff type, but also we investigate their regularity consequences. Apart from precise continuity and Holder continuity criteria, we prove a sharp rearrangement estimate for the potential and prove that its boundedness is equivalent to a one-dimensional Hardy-type inequality. This an efficient tool to transfer regularity from data to solutions.

Speaker: Eleonora Cinti

Title: *Flatness results for stable solutions to some nonlocal problems*

Time slot: *Tuesday, 15:00–15:50*

Abstract:

We study stable critical points to two, closely related, nonlocal functionals: the fractional perimeter and the fractional Allen-Cahn energy. In particular, we establish optimal energy estimates, whose local analogue is still unknown. These estimates, together with other ingredients (such as density estimates and a monotonicity formula) allow to reduce the classification of stable critical points for both functionals to the classification of stable hypercones. These results have been obtained in collaboration with X. Cabré and J. Serra.

Speaker: Cristiana de Filippis

Title: *Nonlocal phenomena in local problems*

Time slot: *Thursday, 11:20–12:10*

Abstract:

In mixed local and nonlocal operators two different orders of differentiation are combined, the simplest model case being $-\Delta + (-\Delta)^s$, for $s \in (0,1)$. Here, the simultaneous presence of a leading local operator, and a lower order fractional one, constitutes the essence of the matter. In this talk I will discuss some recent results that hold for nonlinear mixed problems, focusing in particular on maximal gradient regularity. This is joint work with Giuseppe Mingione (University of Parma).

Speaker: Maria del Mar Gonzalez

Title: *Eigenfunctions for Levy Fokker-Planck equations*

Time slot: *Friday, 10:00–10:50*

Abstract:

When one writes the fractional heat equation in self-similar variables a drift term appears. We study the associated eigenvalue problem for this equation, which has a fractional Laplacian and a first order term under competition. Our main contribution is to give explicit formulas of the fractional analogue of Hermite polynomials. A crucial tool is the Mellin transform, which is essentially the Fourier transform in logarithmic variable and which turns the gradient into multiplication. This is joint work with Hardy Chan, Marco Fontelos and Juncheng Wei.

Speaker: Xavier Fernandez-Real

Title: *Stable cones in the fractional one-phase problem*

Time slot: *Wednesday, 10:00–10:50*

Abstract:

The one-phase problem is the study of minimizers of the functional

$$\mathcal{J}_s(u) = [u]^2_{H^s(B_1)} + |\{u > 0\} \cap B_1|$$

with $s \in (0, 1]$.

In this talk, we will first introduce the basic known results in the classical case, $s = 1$, to then proceed to study the fractional or thin version of the problem.

We will finally present our recent results with X. Ros-Oton, where we found the stability condition for the problem. We then used it to show that any axially symmetric homogeneous stable solution in dimensions $n \leq 5$ is one-dimensional, independently of the parameter $s \in (0, 1)$.

Speaker: Federico Franceschini

Title: *Expansion of the fundamental solution of a second order elliptic operator with analytic coefficients*

Time slot: *Friday, 11:20–12:10*

Abstract:

Let L be a second-order elliptic operator with analytic coefficients defined in $B_1 \subseteq \mathbb{R}^n$. We construct explicitly and canonically a fundamental solution for the operator, i.e., a function $u : B_{r_0} \to \mathbb{R}$ such that $Lu = \delta_0$. As a consequence of our construction, we obtain an expansion of the fundamental solution in homogeneous terms (homogeneous polynomials divided by a power of $|x|$, plus homogeneous polynomials multiplied by $\log(|x|)$ if the dimension n is even) which improves the classical result of F. John (1950). The control we have on the "complexity" of each homogeneous term is optimal and in particular, when L is the Laplace-Beltrami operator of an analytic Riemannian manifold, we recover the construction of the fundamental solution due to K. Kodaira (1949). The main ingredients of the proof are a harmonic decomposition for singular functions and the reduction of the convergence of our construction to a nontrivial estimate on weighted paths on a graph with vertices indexed by $\mathbb{Z}^2$.

Speaker: Tuhin Ghosh

Title: *Inverse problem for the porous medium equation*

Time slot: *Wednesday, 17:10–18:00*

Abstract:

In this talk, we will discuss the inverse boundary value problem associated with the porous medium equation (PME): $\epsilon\partial_t u(t,x) - \nabla\cdot(\gamma\nabla u^m(t,x)) = 0$, with $m > 1$. As its name suggests, the PME can be seen as a model for the flow of a gas through a porous medium, with the function $u(t,x)$ being the density of the gas at time t and position x. The parameters ϵ and γ then depend on the particular gas considered, and also on the properties of the medium. It is a degenerate nonlinear parabolic PDE, also used as a model for phenomena in fields such as plasma physics, and population dynamics. Here we will present a discussion about the unique recovery of the parameters from the relevant boundary Cauchy data.

Speaker: Federico Glaudo

Title: *On the sharp stability of critical points of the Sobolev inequality*

Time slot: *Tuesday, 10:00–10:50*

Abstract:

The unique minimizers of the Sobolev inequality in R^n are known to be the Talenti bubbles, a two parameters (position and concentration) family of functions. As a consequence, the Talenti bubbles solve the associated Euler-Lagrange equation $\Delta u + u^{2^*-1} = 0$ in R^n. If $u : R^n \to R$ is a sum of "almost independent" bubbles, then u "almost solves" the Euler-Lagrange equation, that is $|\Delta u + u^{2^*-1}|_{H^{-1}} << 1$. M. Struwe proved the converse in the 80s, i.e., that if a function u satisfies $|\Delta u + u^{2^*-1}|_{H^{-1}} << 1$ then u is close in H^1 to a sum of almost independent bubbles. With an application to the fast diffusion equation in mind, we will discuss the sharp quantitative stability of Struwe's result. We will present various recent (sharp quantitative) estimates of the distance (in H^1) between u and the manifold of sum of Talenti bubbles with the quantity $|\Delta u + u^{2^*-1}|_{H^{-1}}$. The unexpected and novel feature is that the sharp exponent in these estimates depends on the dimension n.

This talk is based on a joint work with A. Figalli.

Speaker: Gabriele Grillo

Title: *The fractional porous medium equation on classes of noncompact Riemannian manifolds*

Time slot: *Wednesday, 15:00–15:50*

Abstract:

We consider nonlinear evolution equations of porous medium type on noncompact manifolds M, the evolution being driven by the fractional Laplacian on M, such operator being meant in the spectral sense. We prove well-posedness of the problem, in the weak dual sense, for data belonging to a weighted L^1 space where the weight is, roughly speaking, the fractional Green function on the manifold, assumed to exists. Various kind of smoothing effects are proved, depending on geometric assumptions on M. Such assumptions involve: the validity of Faber-Krahn (or Gagliardo-Nirenberg, or Sobolev) inequalities on M, the fact that M is Cartan-Hadamard, namely that M is simply connected with nonpositive curvature, and the condition sec $\leq -c < 0$, sec denoting sectional curvature. Different smoothing effects, stronger as soon as further assumptions are required, for the evolution are proved, according to each of the above conditions. This is a joint work with E. Berchio, M. Bonforte, and M. Muratori.

Speaker: Peio Ibarrondo

Title: *Nonlocal Fast Diffusion Equation on Bounded Domains*

Time slot: *Friday, 12:10–13:00*

Abstract:

We study the Cauchy-Dirichlet Problem for a nonlinear and nonlocal diffusion equation of singular type on bounded domains. Namely, the equations is of the form $\partial_t u = -\mathcal{L}u^m$, where $\mathcal{L}$ is a nonlocal diffusion operator and m belongs to $(0, 1)$. Our results provide a complete basic theory including existence and uniqueness in the biggest class of data known so far, sharp smoothing estimates with weighted and unweighted L^p-norms, and extinction of solutions in finite time. We will compare two strategies to prove smoothing effects: Moser iteration VS Green function method. This is a joint work with M. Bonforte and M. Ispizua.

Speaker: Mikel Ispizua

Title: *A Steklov Version of the Torsional Rigidity*

Time slot: *Tuesday, 17:10–18:00*

Abstract:

Torsional Rigidity is the twisting of an object due to an applied torque. The correct formulation of the torsion problem for a beam with a certain cross section was given by the French mechanician and mathematician A.B. de Saint Venant in the middle of XIXth century. He also stated that the simply connected cross section with maximal torsional rigidity is a circle. We study a boundary variant of the torsional rigidity problem. This corresponds to the sharp constant for the continuous trace embedding of $W^{1,2}(\Omega)$ in $L^1(\partial\Omega)$. We obtain various equivalent variational formulations, present some properties of the state function and obtain some sharp geometric estimates, both for planar simply connected sets and for convex sets in any dimension.

Speaker: Espen Jakobsen

Title: *Existence and uniqueness results for Mean Field Game systems with fractional or nonlinear diffusions*

Time slot: *Wednesday, 12:10–13:00*

Abstract:

Mean Field Games (MFGs) is currently a very active area of research. In these games of large/infinite number of agents, the Nash equilibria can sometimes be described by a coupled system of nonlinear PDEs:

(i) A backward in time Hamilton-Jacobi-Bellman equation for the decision making of the generic agent, and
(ii) a forward in time Fokker-Planck equation for the distribution of agents.

In the presence of noise, both equations include diffusion terms, and in this talk we will discuss existence, uniqueness (and regularity) of solutions in two different cases:

(a) The diffusion is nonlocal/fractional: The noise has long-distance interactions/fat tails, typically modelled by a Levy jump process. In this case the MFG system will be nonlocal. Nonlocal diffusions are sometimes called anomalous diffusions, and are common in e.g. Physics and Finance.
(b) The diffusion is nonlinear: This means that in the underlying controlled SDE, not only the drift is controlled but also the diffusion. In many applications controlled diffusions occur naturally, especially in Finance.

Both results are among the first such extensions to the MFG setting, and include even results for strongly degenerate diffusions (of low order). Ingredients of the proofs include various regularity, compactness and stability results, adjoint methods, heat kernel bounds, semigroup/(very) weak/viscosity solutions/strong formulations, and extensions and improvements of ("Lasry-Lions") fixed point and uniqueness arguments for MFGs. Compared too much of the literature, we work in the whole space and not on the compact torus, which makes proofs more complicated.

Joint work with Olav Ersland, Indranil Chowdhury, and Milosz Krupski.

Speaker: Moritz Kassmann

Title: *Heat kernel bounds for nonlocal operators*

Time slot: *Wednesday, 17:10–18:00*

Abstract:

We report on recent developments in two directions. First, we explain a robustness result for heat kernel bounds for nonlocal operators driven by singular jumping measures. The result is based on a recent paper with Kyung-Youn Kim and Takashi Kumagai. Second, we provide a purely analytical proof of heat kernels upper bounds in the case of jumping measures with rotationally symmetric bounds. We follow the original ideas of Aronson. The result is based on a joint paper with Marvin Weidner.

Speaker: Minhyun Kim

Title: *The Wiener criterion for nonlocal Dirichlet problems*

Time slot: *Thursday, 17:10–18:00*

Abstract:

In this talk, we study the boundary behavior of solutions to the Dirichlet problems for the fractional p-Laplacian-type nonlinear nonlocal operators. We establish a nonlocal counterpart of the Wiener criterion, which characterizes a regular boundary point in terms of the nonlocal nonlinear potential theory. This talk is based on a joint work with Ki-Ahm Lee and Se-Chan Lee.

Speaker: María Medina de la Torre

Title: *Interacting helical traveling waves for the Gross-Pitaevskii equation*

Time slot: *Monday, 15:00–15:50*

Abstract:

In this talk we will construct traveling waves solutions of the 3D Gross-Pitaevskii equation

$$i\partial_t \psi + \Delta\psi + (1 - |\psi|^2)\psi = 0 \quad \text{for} \quad \psi : \mathbb{R} \times \mathbb{R}^3 \to \mathbb{C},$$

whose vortex set is made of an arbitrary number of helical filaments, by means of a Lyapunov-Schmidt reduction argument. These helices are solutions to the Klein-Majda-Damodaran system, supposed to describe the evolution of nearly parallel vortex filaments in ideal fluids.

This is a joint work with J. Dávila (University of Bath), M. del Pino (University of Bath) and Rémy Rodiac (Université Paris-Saclay).

Speaker: Giulia Meglioli

Title: *Nonexistence of solutions to quasilinear parabolic equations with a potential in bounded domains*

Time slot: *Wednesday, 11:20–12:10*

Abstract:

In this seminar we are concerned with nonexistence results for the following class of quasilinear parabolic differential problems

$$\begin{cases} u_t - \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) \geq V\, u^q & \text{in } \Omega \times (0, +\infty) \\ u = 0 & \text{on } \partial\Omega \times (0, +\infty) \\ u = u_0 & \text{in } \Omega \times \{0\}\,; \end{cases} \tag{P}$$

where Ω is an open bounded connected subset of $\mathbb{R}^N$, $p > 1$ and $q > \max\{p-1, 1\}$. Furthermore, we assume that $V \in L^1_{loc}(\Omega \times [0, \infty))$, with $V > 0$ a.e. in $\Omega \times (0, +\infty)$. In particular, we investigate how the behavior of the potential near the boundary of the domain and the power nonlinearity affect the nonexistence of solutions. Particular attention is devoted to the special case of the semilinear parabolic problem, obtained from (P) with $p = 2$, for which we show that the critical rate of growth of the potential near the boundary ensuring nonexistence is sharp. The results presented here are based on a joint work with D.D. Monticelli and F. Punzo, [1].

References

[1] G. Meglioli, D.D. Monticelli, F. Punzo, *Nonexistence of solutions to quasilinear parabolic equations with a potential*, Calc. Var. and PDEs **23** (2022), 61.

Speaker: Matteo Muratori

Title: *The fast diffusion equation on Riemannian manifolds with nonpositive curvature and related elliptic problems*

Time slot: *Tuesday, 15:50–16:40*

Abstract:

We study the fast diffusion equation (FDE) on noncompact Riemannian manifolds with possibly negative Ricci curvature. Existence and uniqueness of solutions in the natural L^1 class of initial data was established by Bonforte, Grillo and Vázquez [J. Evol. Equ. 2008]. However, in the Euclidean space, it is known from the celebrated work of Herrero and Pierre [TAMS 1985] that the FDE is well posed even for initial data that are *locally* integrable. As concerns existence we show that, surprisingly, the same is true on general Riemannian manifolds. If in addition the radial Ricci curvature satisfies a suitable pointwise bound from below (not necessarily constant) related to a stochastic-completeness property of the manifold, we prove that also uniqueness holds, at least in the class of strong solutions. The latter hypothesis can be removed if the initial datum is nonnegative and locally *square* integrable: in that case we can prove that uniqueness holds in the class of nonnegative and *distributional* solutions, a result that to our knowledge was unknown even in the Euclidean space. The required curvature bound is essentially sharp, since for model manifolds it is equivalent to the manifold being *stochastically complete*, and such a property causes uniqueness to fail even in the class of *bounded* solutions. The uniqueness proof relies on crucial auxiliary results, of independent interest, in which we establish nonexistence of nonnegative, nontrivial, distributional subsolutions to certain semilinear elliptic equations with power nonlinearities, in the spirit of Keller and Osserman.

The talk is based on joint works with G. Grillo, K. Ishige and F. Punzo.

Speaker: Simon Nowak

Title: *Sobolev regularity for nonlocal equations with VMO coefficients*

Time slot: *Thursday, 10:00–10:50*

Abstract:

We present some Sobolev regularity results of Calderón-Zygmund-type for nonlocal equations with possibly discontinuous coefficients of VMO-type. While for corresponding local elliptic equations with such irregular coefficients it is in general only possible to obtain higher integrability, in our nonlocal setting we are able to also prove a substantial amount of higher differentiability. Therefore, our results are in some sense of purely nonlocal type, following a recent trend of such results in the literature.

Speaker: Fabio Punzo

Title: *Blow-up for semilinear parabolic equations on Riemannian manifolds and metric graphs*

Time slot: *Wednesday, 15:50–16:40*

Abstract:

This talk is devoted to finite-time blow-up and global in time existence of nonnegative solutions to the semilinear heat equation with a general reaction term $f(u)$, on complete non-compact Riemannian manifolds. In particular, we provide conditions on f and on the manifold ensuring blow-up or global existence. These results have been recently obtained jointly with G. Grillo and G. Meglioli.

Finally, we also see briefly some results, obtained jointly with A. Tesei, about the blow-up of solutions to the semilinear heat equation, posed on metric graphs.

Speaker: Daniel Restrepo

Title: *Uniform stability in the Euclidean Isoperimetric problem for the Allen-Cahn energy*

Time slot: *Monday, 17:10–18:00*

Abstract:

The talk will be mainly focused on a version of the Euclidean isoperimetric for the Allen-Cahn energy. In this joint work with Francesco Maggi, we proved the validity of two fundamental properties of the classical isoperimetric problem for this phase transitions approximation: 1) stability, i.e., the difference in energy between competitors and the global minimum controls quantitatively (and uniformly in the length scale of the phase transition) the distance to minimizers; 2) the only critical points of the associated variational problem (under certain assumptions) are minimizers, i.e., rigidity in the spirit of Alexandrov's theorem.

Speaker: Diana Stan

Title: *The fast p-Laplacian evolution equation. Global Harnack principle and fine asymptotic behavior*

Time slot: *Monday, 12:10–13:00*

Abstract:

We study fine global properties of nonnegative solutions to the Cauchy Problem for the fast p-Laplacian evolution equation on the whole Euclidean space, in the so-called "good fast diffusion range". It is well-known that non-negative solutions behave for large times as B, the Barenblatt (or fundamental) solution, which has an explicit expression. We prove the so-called Global Harnack Principle (GHP), that is, precise global pointwise upper and lower estimates of nonnegative solutions in terms of B. This can be considered the nonlinear counterpart of the celebrated Gaussian estimates for the linear heat equation. To the best of our knowledge, analogous issues for the linear heat equation, do not possess such clear answers, only partial results are known. Also, we characterize the maximal (hence optimal) class of initial data such that the GHP holds, by means of an integral tail condition, easy to check. Finally, we derive sharp global quantitative upper bounds of the modulus of the gradient of the solution, and, when data are radially decreasing, we show uniform convergence in relative error for the gradients. This is joint work with Matteo Bonforte (UAM-ICMAT, Madrid, Spain) and Nikita Simonov (Université Paris-Saclay, CNRS, Univ. Evry, Paris, France).

Speaker: Juan Luis Vázquez

Title: *Equations with p-Laplacian operators of nonlocal type*

Time slot: *Monday, 10:50–11:40*

Abstract:

The p-Laplacian is a model of strongly nonlinear operator with a well-developed elliptic and parabolic theory. In recent years, interest has focused on versions of this model that include nonlocal effects. We describe recent results obtained for the parabolic theory for p finite. Finally, we discuss a nonlocal infinity Laplacian model motivated by a tug-of-war process.

Speaker: Bruno Volzone

Title: *New results on nonlinear aggregation-diffusion equations with Riesz kernels*

Time slot: *Thursday, 12:10–13:00*

Abstract:

One of the archetypical aggregation-diffusion models is the so-called classical parabolic-elliptic Patlak-Keller-Segel (PKS for short) model. This model was classically introduced as the simplest description for chemotactic bacteria movement in which linear diffusion tendency to spread fights the attraction due to the logarithmic kernel interaction in two dimensions. For this model there is a well-defined critical mass. In fact, here a clear dichotomy arises: if the total mass of the system is less than the critical mass, then the long time asymptotics are described by a self-similar solution, while for a mass larger than the critical one, there is finite time blow-up. In this talk we will first give an overview about some results obtained in the papers [1]-[2] concerning the characterization of the stationary states for a nonlinear variant of the PKS model, of the form

$$\partial_t \rho = \Delta \rho^m + \nabla \cdot (\rho \nabla (W * \rho)), \tag{1}$$

being $W \in C^1(\mathbb{R}^d \setminus \{0\})$ a Riesz kernel aggregation, namely $W(x) = c_{d,s}|x|^{2s-d}$ for $s \in (0, d/2)$, in the assumptions of dominated diffusion, i.e. when *i.e.* for $m > 2 - (2s)/d$. In particular, all stationary states of the model are shown to be radially symmetric decreasing and uniquely identified with global minimizers of the associated free energy functionals. In the second part of the talk we will discuss the recent results established in the joint paper [3], in which an addition of a quadratic diffusion term in equation (1) produces a more precise competition with the aggregation term for small s, as they have the same scaling if $s = 0$. We characterize the asymptotic behavior of the stationary states behavior as s goes to zero. Finally, we establish the existence of gradient flow solutions to the evolution problem by applying the JKO scheme.

References

[1] J. A. Carrillo, F. Hoffmann, E. Mainini, B. Volzone. *Ground States in the Diffusion-Dominated Regime*, Calc. Var. Partial Differ. Equ. 57, No. 5, Paper No. 127, 28 p. (2018).

[2] H. Chan, M. González, Y. Huang, E. Mainini, B. Volzone. *Uniqueness of entire ground states for the fractional plasma problem.*, Calc. Var. Partial Differ. Equ. 59, No. 6, Paper No. 195, 41 p. (2020).

[3] Y. Huang, E. Mainini, J. L. Vázquez, B. Volzone. *Nonlinear aggregation-diffusion equations with Riesz potentials*, arXiv:2205.13520 [math.AP] (2022)

Speaker: Marvin Weidner

Title: *Regularity estimates for nonlocal operators related to nonsymmetric forms*

Time slot: *Thursday, 15:00–15:50*

Abstract:
In this talk we consider parabolic equations driven by nonlocal operators with a nonsymmetric jumping kernel. Such operators consist of a fractional order diffusion part and a nonlocal drift part of lower order. Our emphasis is on Hölder regularity estimates and (full) Harnack inequalities for weak solutions to such equations. Moreover, we discuss two-sided estimates for the fundamental solution to the corresponding Cauchy problem. This talk is based on joint work with Moritz Kassmann.

References

[1] Kassmann, Moritz, and Weidner, Marvin. Nonlocal operators related to nonsymmetric forms I: Hölder estimates. *arXiv:2203.07418*, 2022.