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Title: *Sharp well-posedness and blowup results for parabolic systems of the Keller–Segel type*

Abstract:

We study two toy models obtained after a slight modification of the nonlinearity of the usual doubly parabolic Keller–Segel system. For these toy models, both consisting of a system of two parabolic equations, we establish that for data which are, in a suitable sense, smaller than $\tau/(\ln\tau)^3$, where τ is the diffusion parameter in the equation for the chemoattractant, we obtain global solutions. Moreover, for a class of data larger than τ, we obtain the finite time blowup. Thus, our analysis implies that our size condition on the initial data for the global existence of solutions is sharp, for large τ, up to a logarithmic factor.

We consider parabolic systems (TM) and (TM') below, depending on a diffusion parameter $\tau > 0$:

$$\begin{cases} u_t = \Delta u - u\Delta\varphi, \\ \tau\varphi_t = \Delta\varphi + u, & x \in \mathbb{R}^d,\ t > 0, \\ u(0) = u_0, \quad \varphi(0) = \varphi_0, \end{cases} \tag{TM}$$

and

$$\begin{cases} u_t = \Delta u + (\Delta\varphi)^2, \\ \tau\varphi_t = \Delta\varphi + u, & x \in \mathbb{R}^d,\ t > 0, \\ u(0) = u_0, \quad \varphi(0) = \varphi_0, \end{cases} \tag{TM'}$$

We introduce these model systems in order to show the influence of the parameter τ in the second equation (a linear nonhomogeneous heat equation) on the size of admissible initial data leading to global-in-time solutions, and to finite time blowup, respectively.

Our main motivation is to analyze those issues for the parabolic-parabolic Keller–Segel system describing chemotaxis,

$$\begin{cases} u_t = \Delta u - \nabla\cdot(u\nabla\varphi), \\ \tau\varphi_t = \Delta\varphi + u, & x \in \mathbb{R}^d,\ t > 0. \\ u(0) = u_0, \quad \varphi(0) = \varphi_0, \end{cases} \tag{PP}$$

We studied the global existence of solutions for (PP) in the recent paper, for initial data inside balls centered at the origin and of radius $R = R(\tau) > 0$, in different functional spaces. The optimal value of $R(\tau)$ expresses the best possible size conditions on the data preventing blowup. Interestingly, the asymptotic behavior of the radius $R(\tau)$, as $\tau \to +\infty$, seems to be sensitive with respect to the choice of the norm. But we could not completely achieve our program for (PP), because of the lack of results on finite time blowup.

In the present paper, for the models (TM) and (TM'), we will be able to achieve our program, by providing nearly sharp size conditions for both the global existence and the blowup. We stress the fact that both toy models have the same scaling properties as (PP), so their analysis should bring new light on the mathematical analysis of the Keller–Segel system.