Speaker: Dariusz Wrzosek

Title: *Pursuit-evasion dynamics in prey-predator models*

Abstract:

We study a pursuit-evasion predator-prey system

$$\begin{cases} P_t = d_P \Delta P - \xi \nabla \cdot P \ \nabla N + f(P, N)\,, \\ N_t = d_N \Delta N + \chi \nabla \cdot N \ \nabla W + g(N, P)\,, \\ W_t = d_W \Delta W + \gamma P - \mu W\,, \end{cases} \tag{1}$$

defined on a smooth boundary $\Omega \subset \mathbb{R}^n$ with no-flux boundary condition and initial conditions from the Sobolev space $W^{1,p}(\Omega)\,, p > n$. The variables P and N denote predator and prey densities while W is the density of a chemical secreted by the predators. The reaction functions f and g describe predator-prey interactions and birth/death processes, $d_P\,, d_N\,, d_W > 0$ are diffusion constants, $\xi\,, \eta > 0$ are taxis sensitivity parameters, γ and μ are rate coefficients. The taxis term in the first equation describes direct prey taxis i.e. the movement of predators towards the density gradient of prey (pursuit) while the second equation represents situation in which the prey senses not the presence of predators themselves but rather their odor, a diffusive chemical with density W so that the prey use evasive strategy moving in the opposite direction with respect to the gradient of W. We study the existence of global-in-time classical solutions to the system (1) and formation of space-time patterns for the range of parameters when a space-homogeneous coexistence steady state loses its stability. For realistic assumptions on f and g, as in Bazykin's model, the global solutions exist for space dimension $n = 1$ while for $n = 2$ numerical solutions indicate the blow-up of solutions in finite time (see [1, 2]). The blow-up is a cumulative effect of both taxis mechanisms (i.e $\xi\,, \chi > 0$) as neither of taxis alone can lead to the blow-up for the space dimension $n = 2$. We also propose a possibly minimal modification of (1) which warrants prevention of blow-up formation in finite time.

References

[1] P. Mishra, D. Wrzosek, Repulsive chemotaxis and predator evasion in predator prey models with diffusion and prey-taxis, *Mathematical Models and Methods in Applied Sciences*, 32 (1):1-42, 2022.

[2] P. Mishra, D. Wrzosek, Pursuit-evasion dynamics for Bazykin-type predator-prey model with indirect predator taxis, *Journal of Differential Equations*, 361:391-416, 2023.