Speaker: Håkon A. Hoel

Title: *Higher-order adaptive methods for exit times of diffusion processes*

Abstract:

The Feynman–Kac formula provides a connection between domain-exit and boundary-reflection properties of stochastic differential equations (SDE) and parabolic partial differential equations. The SDE viewpoint is particularly interesting for numerical methods, as it can be used with Monte Carlo methods to overcome the curse of dimensionality when solving high-dimensional PDE. This however hinges on having an efficient numerical method for simulating the exit times of SDEs. Since exit times of diffusion processes are very sensitive to perturbations in initial conditions, it is challenging to construct such numerical methods.

This talk presents a high-order method with adaptive timestepping for strong approximations of exit times. The method employs a high-order Itô–Taylor scheme for simulating SDE paths and carefully decreases the step size in the numerical integration as the diffusion process approaches the domain's boundary. These techniques complement each other well: adaptive time-stepping improves the accuracy of the exit time by reducing the overshoot out of the domain, and high-order schemes improve the state approximation of the diffusion process, which is useful feedback to control the step size. We will also briefly discuss an ongoing extension of the numerical method to reflected diffusion SDEs (relating to PDE with Neumann boundary conditions).