Speaker: Ola Mæhlen

Title: *Transport equations for Osgood velocity fields*

Abstract:

In the smooth setting, there is a direct connection between solutions of the transport equation (a PDE) and the trajectories of the underlying velocity field (a family of ODEs): Solutions of the PDE are transported along the trajectories of the ODE. Outside the smooth setting, the connection is less clear, and the PDE or the family of ODEs may be ill-posed. Still, there are manageable cases. Most prominently, is the work of DiPerna–Lions who require that the velocity field enjoys some Sobolev regularity and is of bounded divergence: These are natural assumptions on the PDE, whose resulting well-posedness give an analogous well-posedness theory for the underlying family of ODEs.

However, the family of ODEs is also well-posed under the Osgood condition which is to demand a continuity modulus (for the velocity field) whose reciprocal is not integrable at zero. This condition is independent of the criteria of the DiPerna–Lions theory, and while it is natural for the ODEs, it is not so immediate what it implies for the PDE. In this talk we will show that, granted some extra assumptions, the Osgood condition give rise to a well-posed transport equation, and that the unique solutions enjoy the transport property (just as in the smooth setting). The approach relies on interpreting some of the (seemingly ill-posed) integrals in the weak formulation as Riemann–Stieltjes integrals. From there, a novel estimate in the BV-setting is derived (proving uniqueness of BV-solutions) and then generalized to a p-variation setting through a new decomposition formula that resembles, but is different from, Littlewood–Paley decomposition. This is joint work with Ulrik Skre Fjordholm (University of Oslo).