Speaker: Artur Rutkowski

Title: Caloric functions for the fractional Laplacian in Lipschitz sets

Abstract:

We investigate nonnegative caloric functions for the fractional Laplacian, i.e., the solutions of the fractional heat equation, in bounded Lipschitz sets. Our primary definition is via the mean value property for the space-time isotropic α -stable process, which can be conveniently expressed in the language of the Dirichlet heat kernel of the fractional Laplacian. With this definition we give an integral representation for caloric functions – each of them can be uniquely determined by its initial values, exterior values, and a certain measure on the boundary. We also show that our caloric functions coincide with distributional solutions and we give a sufficient condition on the data under which the caloric function is a classical solution to the fractional heat equation. The results were obtained in collaboration with G. Armstrong and K. Bogdan.